

Probability

INTRODUCTION Historically, the theory of probability began to develop with the study of games of chance such as roulette and cards. Apart from games, uncertainty also prevails in other walks of life such as business, economy, and even in day-to-day activities.

Probability is a concept which numerically measures the degree of uncertainty and, therefore, of certainty of the occurrence of an event.

Before defining some terms related to probability, we shall give certain concepts used therein.

EXPERIMENT *An operation which can produce some well-defined outcomes is known as an experiment.*

In science or engineering if we perform an experiment and repeat it under identical conditions, we get almost the same result every time. Such experiments are called *deterministic experiments*.

But, there are experiments, which when repeated under identical conditions, do not produce the same outcome every time. For example, if we toss a fair coin, we may get a head or a tail. Now, if we make further trials, i.e., toss the coin again and again, the outcome of each trial depends on chance, and it is not the same each time. Sometimes the head appears and sometimes, the tail. Such experiments are called *random experiments*.

RANDOM EXPERIMENT *If in each trial of an experiment, conducted under identical conditions, the outcome is not unique, but may be any of the several possible outcomes then such an experiment is known as a random experiment.*

In a random experiment, the outcome of each trial depends on chance, which is beyond our control, and as such it cannot be predicted with certainty.

EXAMPLES Tossing a fair coin, rolling an unbiased die, drawing a card from a well-shuffled pack of cards are all examples of random experiments.

NOTE 1 A *die* is a solid cube, the six faces of which are marked with 1, 2, 3, 4, 5 and 6 dots respectively. In throwing a die, the outcome is the number of dots on the uppermost face.
The plural of die is *dice*.



NOTE 2 A pack of cards consists of 52 cards in four suits, called *spades*, *clubs*, *hearts* and *diamonds*. Out of these, spades and clubs are black-faced cards, while hearts and diamonds are red-faced cards. The aces, kings, queens and jacks are known as *face cards*.

SAMPLE SPACE The set of all possible outcomes in a random experiment is called a sample space and it is generally denoted by S .

Each element of a sample space is called a *sample point*.

EXAMPLES OF SAMPLE SPACES

EXAMPLE 1 List the sample space in tossing a fair coin.

SOLUTION In tossing a fair coin, there are two possible outcomes, namely, head (H) and tail (T).

Hence, the sample space in this experiment is given by

$$S = \{H, T\}.$$

EXAMPLE 2 List the sample space in throwing a die.

SOLUTION When we throw a die, it can result in any of the six numbers, namely, 1, 2, 3, 4, 5, 6.

$\therefore$ the sample space is $S = \{1, 2, 3, 4, 5, 6\}$.

EXAMPLE 3 Two coins are tossed together. List the sample space.

SOLUTION When two coins are tossed together, the sample space is

$$S = \{HT, TH, HH, TT\}.$$

Here, HT shows a head on the first coin and a tail on the second. Similarly, TH means a tail on the first and a head on the second; HH means a head on each, and TT means a tail on each.

EXAMPLE 4 Three coins are tossed simultaneously. List the sample space for the event. [CBSE 1999]

SOLUTION When three coins are tossed simultaneously, the sample space is given by

$$S = \{HHH, HHT, HTH, THH, HTT, TTH, THT, TTT\}.$$

EXAMPLE 5 List the sample space in a simultaneous toss of a die and a coin.

SOLUTION Clearly, the sample space is given by

$$S = \{(1, H), (2, H), (3, H), (4, H), (5, H), (6, H), (1, T), (2, T), (3, T), (4, T), (5, T), (6, T)\}.$$

EXAMPLE 6 Two well-balanced dice are rolled and the numbers that turn up are observed. Determine the sample space. [CBSE 1999]

SOLUTION When two dice are rolled, there are $(6 \times 6) = 36$ outcomes. The set of all these outcomes is the sample space, given by



$$S = \{(1, 1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1), \\ (1, 2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2), \\ (1, 3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3), \\ (1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4), \\ (1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), \\ (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)\}.$$

EXAMPLE 7 From a group of 3 boys and 2 girls, we select two children. What would be the sample space for this experiment?

SOLUTION Let us name the boys as B_1 , B_2 and B_3 , and the girls as G_1 and G_2 . Then the sample space is given by

$$S = \{B_1B_2, B_1B_3, B_1G_1, B_1G_2, B_2B_3, B_2G_1, B_2G_2, B_3G_1, B_3G_2, G_1G_2\}.$$

EXAMPLE 8 A coin is tossed twice. If the second throw results in a tail then a die is thrown. Describe the sample space.

SOLUTION Clearly, the sample space is given by

$$S = \{HH, TH, HT1, HT2, HT3, HT4, HT5, HT6, TT1, TT2, TT3, \\ TT4, TT5, TT6\}.$$

EXAMPLE 9 A coin is tossed. If it shows a tail, we draw a ball from a box which contains 2 red and 3 black balls; if it shows a head, we throw a die. Find the sample space for this experiment.

SOLUTION Let the red balls be named as R_1 and R_2 , and the black balls be named as B_1 , B_2 and B_3 .

Then, the sample space is given by

$$S = \{TR_1, TR_2, TB_1, TB_2, TB_3, H1, H2, H3, H4, H5, H6\}.$$

EXAMPLE 10 An experiment consists of rolling a die and then tossing a coin once if the number on the die is even. If the number on the die is odd, the coin is tossed twice. Write the sample space for this experiment.

SOLUTION Clearly, the sample space is given by

$$S = \{2H, 2T, 4H, 4T, 6H, 6T, 1HH, 1HT, 1TH, 1TT, 3HH, 3HT, \\ 3TH, 3TT, 5HH, 5HT, 5TH, 5TT\}.$$

EXAMPLE 11 A bag contains 4 identical red balls and 3 identical black balls. The experiment consists of drawing one ball, then putting it into the bag and again drawing a ball. What are the possible outcomes of this experiment?

SOLUTION Let us denote a red ball by R and a black ball by B . Then, clearly the sample space is given by

$$S = \{RR, RB, BR, BB\}.$$

Now, we define some terms related to probability.

EVENT Every subset of a sample space is called an event.

EXAMPLE In a single throw of a die, we have sample space $S = \{1, 2, 3, 4, 5, 6\}$.



The event of getting a prime number is given by
 $E = \{2, 3, 5\}$.

Clearly, $E \subseteq S$.

IMPOSSIBLE EVENT Let S be a sample space.

Since $\phi \subset S$, ϕ is an event, called an *impossible event*.

SURE EVENT Let S be a sample space.

Since $S \subseteq S$, S is an event, called a *sure event*.

EXAMPLE In a throw of a die, we have

sample space $S = \{1, 2, 3, 4, 5, 6\}$.

Let E_1 = event of getting a number less than 1.

And, E_2 = event of getting a number less than 7.

Clearly, no outcome can be less than 1.

$\therefore E_1$ is an *impossible event*.

Also, each outcome is a number less than 7.

$\therefore E_2$ is a *sure event*.

SIMPLE EVENT An event containing only a single element of the sample space is called a *simple* or an *elementary event*.

COMPOUND EVENT An event which is not simple is called a *compound* or *composite* or *mixed event*.

EXAMPLE In a simultaneous toss of two coins, we have

sample space $S = \{HH, HT, TH, TT\}$.

Then E_1 = event of getting a tail on both the coins = $\{TT\}$,
is a simple event.

And, E_2 = event of getting at least 1 tail = $\{HT, TH, TT\}$,
is a compound event.

MUTUALLY EXCLUSIVE EVENTS Two events E_1 and E_2 are said to be mutually exclusive if $E_1 \cap E_2 = \phi$.

However, if $E_1 \cap E_2 \neq \phi$ then E_1 and E_2 are called *compatible events*.

EXAMPLES (i) In throwing a die, we have $S = \{1, 2, 3, 4, 5, 6\}$.

Let E_1 = event of getting a number less than 3.

And, E_2 = event of getting a number more than 4.

Then $E_1 = \{1, 2\}$ and $E_2 = \{5, 6\}$.

Clearly, $E_1 \cap E_2 = \phi$.

Hence, E_1 and E_2 are mutually exclusive.

(ii) In a simultaneous toss of two coins, we have

$S = \{HH, HT, TH, TT\}$.



Let E_1 = event of getting a head on the first coin = $\{HH, HT\}$.
 And, E_2 = event of getting a tail on the second coin = $\{HT, TT\}$.
 Clearly, $E_1 \cap E_2 \neq \phi$.
 Hence, E_1 and E_2 are compatible events.

EXAMPLE 1 Two dice are rolled. Let A, B, C be the events of getting a sum of 2, a sum of 3 and a sum of 4 respectively. Then, show that

- (i) A is a simple event
- (ii) B and C are compound events
- (iii) A and B are mutually exclusive

SOLUTION Clearly, we have

$$A = \{(1, 1)\}, B = \{(1, 2), (2, 1)\}, \text{ and } C = \{(1, 3), (3, 1), (2, 2)\}.$$

- (i) Since A consists of a single sample point, it is a simple event.
- (ii) Since both B and C contain more than one sample point, each one of them is a compound event.
- (iii) Since $A \cap B = \phi$, A and B are mutually exclusive.

EXAMPLE 2 From a group of 2 boys and 3 girls, two children are selected at random. Describe the events:

- (i) A = event that both the selected children are girls
- (ii) B = event that the selected group consists of one boy and one girl
- (iii) C = event that at least one boy is selected

Which pairs of events are mutually exclusive?

SOLUTION Let us name the boys as B_1 and B_2 , and the girls as G_1, G_2 and G_3 .
 Then

$$S = \{B_1B_2, B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, G_1G_2, G_1G_3, G_2G_3\}$$

We have

- (i) $A = \{G_1G_2, G_1G_3, G_2G_3\}$
- (ii) $B = \{B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3\}$
- (iii) $C = \{B_1G_1, B_1G_2, B_1G_3, B_2G_1, B_2G_2, B_2G_3, B_1B_2\}$

Clearly, $A \cap B = \phi$ and $A \cap C = \phi$.

Hence, (A, B) and (A, C) are mutually exclusive events.

MUTUALLY EXCLUSIVE AND EXHAUSTIVE SYSTEM OF EVENTS Let $E_1, E_2, \dots, E_n$ be subsets of a sample space S . Then, we say that

the events $E_1, E_2, \dots, E_n$ form a mutually exclusive and exhaustive system if

- (i) $E_i \cap E_j \neq \phi$ for $i \neq j$, and (ii) $E_1 \cup E_2 \cup \dots \cup E_n = S$.

EXAMPLE Suppose we draw a card from a well-shuffled pack of 52 cards.

Let E_1, E_2, E_3 and E_4 be the events of drawing a spade, drawing a club, drawing a heart and drawing a diamond respectively.



As the card drawn is necessarily one of the four types, one of these events is sure to occur.

When one of these events occurs then none of the others occur.

Thus, E_1, E_2, E_3, E_4 form a mutually exclusive and exhaustive system of events.

EXAMPLE Two dice are rolled. A is the event that the sum of the numbers shown on the two dice is 5, and B is the event that at least one of the dice shows up a 3. Are the two events (i) mutually exclusive, (ii) exhaustive? Give arguments in support of your answer. [CBSE 2001C]

SOLUTION When two dice are rolled, we have $n(S) = (6 \times 6) = 36$.

Now, $A = \{(1, 4), (2, 3), (4, 1), (3, 2)\}$, and

$B = \{(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (1, 3), (2, 3), (4, 3), (5, 3), (6, 3)\}$.

(i) $A \cap B = \{(2, 3), (3, 2)\} \neq \phi$.

Hence, A and B are not mutually exclusive.

(ii) Also, $A \cup B \neq S$.

$\therefore A$ and B are not exhaustive events.

INDEPENDENT EVENTS Two events E_1 and E_2 are said to be independent if the occurrence of one does not depend upon the occurrence of the other.

If the two events are not independent, they are known as *dependent events*.

EXAMPLE Suppose we toss two unbiased coins.

Let E_1 = event of getting a head on the first coin.

And, E_2 = event of getting a head on the second coin.

Clearly, the occurrence of a head on the second coin does not depend upon the occurrence of a head on the first coin.

$\therefore E_1$ and E_2 are independent events.

OCCURRENCE OF AN EVENT In a random experiment, let S be the sample space and let $E \subseteq S$. Then, E is an event.

Let w be an outcome of a trial.

If $w \in E$, we say that the event E has occurred.

If $w \notin E$, we say that the event E has not occurred.

EXAMPLE In a throw of a die, we have $S = \{1, 2, 3, 4, 5, 6\}$.

Let E be the event of getting an even number.

Then $E = \{2, 4, 6\}$.

In a trial, let the outcome be 4.

Since $4 \in E$, we say that the event E has occurred.

In another trial, let the outcome be 5.

Since $5 \notin E$, in this case, the event E has not occurred.



EQUALLY LIKELY EVENTS A given number of events are said to be equally likely if none of them is expected to occur in preference to the others.

EXAMPLE If we roll an unbiased die, each outcome is equally likely to happen.

If, however, a die is so formed that a particular face occurs most often then the die is biased. In this case, the outcomes are not equally likely to happen.

COMPLEMENTARY EVENTS In a random experiment, let S be the sample space and let E be an event. Then $E \subseteq S$.

Now, $E \subseteq S \Rightarrow E^c \subseteq S$.

Thus, E^c is also an event, called the *complement of E* .

We denote it by E^c or E' or $\bar{E}$ and call it *not E* .

Clearly, $\bar{E}$ occurs when E does not occur.

And, E occurs when $\bar{E}$ does not occur.

Thus, in a trial, one of two events, E or $\bar{E}$, is sure to occur.

ALGEBRA OF EVENTS In a random experiment, let S be the sample space.

Let $E \subseteq S$ and $F \subseteq S$.

Then, E as well as F is an event.

We say that

- (i) $(E \cap F)$ is an event that occurs only when each one of E and F occurs
- (ii) $(E \cup F)$ is an event that occurs only when at least one of E and F occurs
- (iii) $\bar{E}$ is an event that occurs only when E does not occur

PROBABILITY OF AN EVENT

INTRODUCTION Suppose a bag contains 90 red and 10 white balls, which are similar in shape and size.

If the balls are mixed thoroughly and then one ball is drawn at random, it will be either red or white.

Clearly, *the ball drawn is more likely to be red than white.*

We express it by saying that the event of drawing a red ball is more probable than the event of drawing a white ball.

To every event associated with a random experiment, we try to attach a numerical value, called its *probability*, in such a manner that for any two events, the event which is more likely to happen has a higher probability.



PROBABILITY OF AN EVENT In a random experiment, let S be the sample space and let $E \subseteq S$. Then, E is an event.

The probability of occurrence of E is defined as

$$\begin{aligned} P(E) &= \frac{\text{number of outcomes favourable to occurrence of } E}{\text{number of all possible outcomes}} \\ &= \frac{\text{number of distinct elements in } E}{\text{number of distinct elements in } S} \\ &= \frac{n(E)}{n(S)} . \\ \therefore P(E) &= \frac{n(E)}{n(S)} . \end{aligned}$$

SOLVED EXAMPLES

EXAMPLE 1 A coin is tossed once. Find the probability of getting a head.

SOLUTION When a coin is tossed once, the sample space is given by $S = \{H, T\}$.

Let E be the event of getting a head.

Then, $E = \{H\}$.

$\therefore n(E) = 1$ and $n(S) = 2$

$$\Rightarrow P(\text{getting a head}) = P(E) = \frac{n(E)}{n(S)} = \frac{1}{2} .$$

EXAMPLE 2 Two coins are tossed once. Find the probability of

- | | |
|-----------------------|--------------------------------|
| (i) getting 2 heads | (ii) getting at least 1 head |
| (iii) getting no head | (iv) getting 1 head and 1 tail |

SOLUTION When two coins are tossed once, the sample space is given by $S = \{HH, HT, TH, TT\}$ and, therefore, $n(S) = 4$.

(i) Let E_1 = event of getting 2 heads. Then,

$E_1 = \{HH\}$ and, therefore, $n(E_1) = 1$.

$$\therefore P(\text{getting 2 heads}) = P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{4} .$$

(ii) Let E_2 = event of getting at least 1 head. Then,

$E_2 = \{HT, TH, HH\}$ and, therefore, $n(E_2) = 3$.

$$\therefore P(\text{getting at least 1 head}) = P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{4} .$$

(iii) Let E_3 = event of getting no head. Then,

$E_3 = \{TT\}$ and, therefore, $n(E_3) = 1$.

$$\therefore P(\text{getting no head}) = P(E_3) = \frac{n(E_3)}{n(S)} = \frac{1}{4} .$$



(iv) Let E_4 = event of getting 1 head and 1 tail. Then,

$$E_4 = \{HT, TH\} \text{ and, therefore, } n(E_4) = 2$$

$$\therefore P(\text{getting 1 head and 1 tail}) = P(E_4) = \frac{n(E_4)}{n(S)} = \frac{2}{4} = \frac{1}{2}.$$

EXAMPLE 3 Three unbiased coins are tossed once. What is the probability of getting

- (i) all heads? (ii) two heads? (iii) one head?
(iv) at least 1 head? (v) at least 2 heads?

SOLUTION In tossing three coins, the sample space is given by

$$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}.$$

And, therefore, $n(S) = 8$.

(i) Let E_1 = event of getting all heads. Then,

$$E_1 = \{HHH\} \text{ and, therefore, } n(E_1) = 1.$$

$$\therefore P(\text{getting all heads}) = P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{8}.$$

(ii) Let E_2 = event of getting 2 heads. Then,

$$E_2 = \{HHT, HTH, THH\} \text{ and, therefore, } n(E_2) = 3.$$

$$\therefore P(\text{getting 2 heads}) = P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{8}.$$

(iii) Let E_3 = event of getting 1 head. Then,

$$E_3 = \{HTT, THT, TTH\} \text{ and, therefore, } n(E_3) = 3.$$

$$\therefore P(\text{getting 1 head}) = P(E_3) = \frac{n(E_3)}{n(S)} = \frac{3}{8}.$$

(iv) Let E_4 = event of getting at least 1 head. Then,

$$E_4 = \{HTT, THT, TTH, HHT, HTH, THH, HHH\}.$$

And, therefore, $n(E_4) = 7$.

$$\therefore P(\text{getting at least 1 head}) = P(E_4) = \frac{n(E_4)}{n(S)} = \frac{7}{8}.$$

(v) Let E_5 = event of getting at least 2 heads. Then,

$$E_5 = \{HHT, HTH, THH, HHH\} \text{ and, therefore, } n(E_5) = 4.$$

$$\therefore P(\text{getting at least 2 heads}) = P(E_5) = \frac{n(E_5)}{n(S)} = \frac{4}{8} = \frac{1}{2}.$$

EXAMPLE 4 A die is tossed once. What is the probability of getting

- (i) the number 4? (ii) an even number?
(iii) a number less than 5? (iv) a number greater than 4?
(v) the number 8? (vi) a number less than 8?

SOLUTION In tossing a die once, the sample space is given by

$$S = \{1, 2, 3, 4, 5, 6\} \text{ and, therefore, } n(S) = 6.$$



- (i) Let E_1 = event of getting the number 4.
Then, $E_1 = \{4\}$ and, therefore, $n(E_1) = 1$.
 $\therefore P(\text{getting the number 4}) = P(E_1) = \frac{n(E_1)}{n(S)} = \frac{1}{6}$.
- (ii) Let E_2 = event of getting an even number. Then,
 $E_2 = \{2, 4, 6\}$ and, therefore, $n(E_2) = 3$.
 $\therefore P(\text{getting an even number}) = P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{6} = \frac{1}{2}$.
- (iii) Let E_3 = event of getting a number less than 5. Then,
 $E_3 = \{1, 2, 3, 4\}$ and, therefore, $n(E_3) = 4$.
 $\therefore P(\text{getting a number less than 5}) = P(E_3) = \frac{n(E_3)}{n(S)} = \frac{4}{6} = \frac{2}{3}$.
- (iv) Let E_4 = event of getting a number greater than 4. Then,
 $E_4 = \{5, 6\}$ and, therefore, $n(E_4) = 2$.
 $\therefore P(\text{getting a number greater than 4})$
 $= P(E_4) = \frac{n(E_4)}{n(S)} = \frac{2}{6} = \frac{1}{3}$.
- (v) Let E_5 = event of getting the number 8.
Since no face of the die is marked with the number 8,
we have $E_5 = \phi$ and, therefore, $n(E_5) = 0$.
 $\therefore P(\text{getting the number 8}) = P(E_5) = \frac{n(E_5)}{n(S)} = \frac{0}{6} = 0$.
- (vi) Let E_6 = event of getting a number less than 8. Then,
 $E_6 = \{1, 2, 3, 4, 5, 6\}$ and, therefore, $n(E_6) = 6$.
 $\therefore P(\text{getting a number less than 8}) = P(E_6) = \frac{n(E_6)}{n(S)} = \frac{6}{6} = 1$.

EXAMPLE 5 In a single throw of two dice, find the probability of obtaining 'a total of 8'.

SOLUTION We know that in a single throw of two dice, the total number of possible outcomes is $(6 \times 6) = 36$.

Let S be the sample space. Then, $n(S) = 36$.

Let E = event of getting a total of 8. Then,

$E = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$ and, therefore, $n(E) = 5$.

$$\therefore P(\text{getting a total of 8}) = P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$$

EXAMPLE 6 Two dice are thrown simultaneously. Find the probability of getting

- (i) a doublet
- (ii) an even number as the sum
- (iii) a prime number as the sum
- (iv) a multiple of 3 as the sum



- (v) a total of at least 10
 (vi) a doublet of even numbers
 (vii) a multiple of 2 on one die and a multiple of 3 on the other die

SOLUTION We know that in a single throw of two dice, the total number of possible outcomes is $(6 \times 6) = 36$.

Let S be the sample space. Then, $n(S) = 36$.

- (i) Let E_1 = event of getting a doublet. Then,
 $E_1 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$.
 $\therefore n(E_1) = 6$.
 $\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$.
- (ii) Let E_2 = event of getting an even number as the sum. Then,
 $E_2 = \{(1, 1), (1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 3), (3, 1),$
 $(3, 5), (4, 2), (4, 4), (4, 6), (5, 1), (5, 3), (5, 5), (6, 2),$
 $(6, 4), (6, 6)\}$.
 $\therefore n(E_2) = 18$.
 $\therefore P(E_2) = \frac{n(E_2)}{n(S)} = \frac{18}{36} = \frac{1}{2}$.
- (iii) Let E_3 = event of getting a prime number as the sum. Then,
 $E_3 = \{(1, 1), (1, 2), (1, 4), (1, 6), (2, 1), (2, 3), (2, 5), (3, 2),$
 $(3, 4), (4, 1), (4, 3), (5, 2), (5, 6), (6, 1), (6, 5)\}$.
 $\therefore n(E_3) = 15$.
 $\therefore P(E_3) = \frac{n(E_3)}{n(S)} = \frac{15}{36} = \frac{5}{12}$.
- (iv) Let E_4 = event of getting a multiple of 3 as the sum. Then,
 $E_4 = \{(1, 2), (1, 5), (2, 1), (2, 4), (3, 3), (3, 6), (4, 2), (4, 5),$
 $(5, 1), (5, 4), (6, 3), (6, 6)\}$.
 $\therefore n(E_4) = 12$.
 $\therefore P(E_4) = \frac{n(E_4)}{n(S)} = \frac{12}{36} = \frac{1}{3}$.
- (v) Let E_5 = event of getting a total of at least 10. Then,
 E_5 = event of getting a total of 10 or 11 or 12
 $= \{(4, 6), (5, 5), (5, 6), (6, 4), (6, 5), (6, 6)\}$.
 $\therefore n(E_5) = 6$.
 $\therefore P(E_5) = \frac{n(E_5)}{n(S)} = \frac{6}{36} = \frac{1}{6}$.
- (vi) Let E_6 = event of getting a doublet of even numbers. Then,
 $E_6 = \{(2, 2), (4, 4), (6, 6)\}$.



$$\therefore n(E_6) = 3.$$

$$\therefore P(E_6) = \frac{n(E_6)}{n(S)} = \frac{3}{36} = \frac{1}{12}.$$

(vii) Let E_7 = event of getting a multiple of 2 on one die and a multiple of 3 on the other die. Then,

$$E_7 = \{(2, 3), (2, 6), (4, 3), (4, 6), (6, 3), (6, 6), (3, 2), (3, 4), (3, 6), (6, 2), (6, 4)\}.$$

$$\therefore n(E_7) = 11.$$

$$\therefore P(E_7) = \frac{n(E_7)}{n(S)} = \frac{11}{36}.$$

EXAMPLE 7 20 cards are numbered from 1 to 20. One card is then drawn at random.

What is the probability that the number on the card drawn is

- (i) a prime number? (ii) an odd number?
 (iii) a multiple of 5? (iv) not divisible by 3?

SOLUTION Clearly, the sample space is given by

$$S = \{1, 2, 3, 4, 5, \dots, 19, 20\} \text{ and, therefore, } n(S) = 20.$$

(i) Let E_1 = event of getting a prime number. Then,

$$E_1 = \{2, 3, 5, 7, 11, 13, 17, 19\} \text{ and, therefore, } n(E_1) = 8.$$

$$\therefore P(\text{getting a prime number}) = P(E_1) = \frac{n(E_1)}{n(S)} = \frac{8}{20} = \frac{2}{5}.$$

(ii) Let E_2 = event of getting an odd number. Then,

$$E_2 = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\} \text{ and, therefore, } n(E_2) = 10.$$

$$\therefore P(\text{getting an odd number}) = P(E_2) = \frac{n(E_2)}{n(S)} = \frac{10}{20} = \frac{1}{2}.$$

(iii) Let E_3 = event of getting a multiple of 5. Then,

$$E_3 = \{5, 10, 15, 20\} \text{ and, therefore, } n(E_3) = 4.$$

$$\therefore P(\text{getting a multiple of 5}) = P(E_3) = \frac{n(E_3)}{n(S)} = \frac{4}{20} = \frac{1}{5}.$$

(iv) Let E_4 = event of getting a number which is not divisible by 3.

$$\text{Then, } E_4 = \{1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20\} \text{ and so, } n(E_4) = 14.$$

$$\therefore P(\text{getting a number which is not divisible by 3})$$

$$= P(E_4) = \frac{n(E_4)}{n(S)} = \frac{14}{20} = \frac{7}{10}.$$

EXAMPLE 8 From a well-shuffled deck of 52 cards, a card is drawn at random. Find the probability of getting

- (i) an ace (ii) a heart (iii) an eight of hearts
 (iv) a club (v) a red card (vi) a face card
 (vii) a diamond (viii) a jack (ix) a black card



SOLUTION Let S denote the sample space. Then, $n(S) = 52$.

(i) Let E_1 = event of drawing an ace.

Since, the number of all aces is 4, so $n(E_1) = 4$.

$$\therefore P(\text{getting an ace}) = P(E_1) = \frac{n(E_1)}{n(S)} = \frac{4}{52} = \frac{1}{13}.$$

(ii) Let E_2 = event of getting a heart. Then, $n(E_2) = 13$.

$$\therefore P(\text{getting a heart}) = P(E_2) = \frac{n(E_2)}{n(S)} = \frac{13}{52} = \frac{1}{4}.$$

(iii) Let E_3 = event of getting an eight of hearts. Then, $n(E_3) = 1$.

$$\therefore P(\text{getting an eight of hearts}) = P(E_3) = \frac{n(E_3)}{n(S)} = \frac{1}{52}.$$

(iv) Let E_4 = event of getting a club. Then, $n(E_4) = 13$.

$$\therefore P(\text{getting a club}) = P(E_4) = \frac{n(E_4)}{n(S)} = \frac{13}{52} = \frac{1}{4}.$$

(v) Let E_5 = event of getting a red card. Then, $n(E_5) = 26$.

$$\therefore P(\text{getting a red card}) = P(E_5) = \frac{n(E_5)}{n(S)} = \frac{26}{52} = \frac{1}{2}.$$

(vi) Let E_6 = event of getting a face card. Then, $n(E_6) = 16$.

$$\therefore P(\text{getting a face card}) = P(E_6) = \frac{n(E_6)}{n(S)} = \frac{16}{52} = \frac{4}{13}.$$

(vii) Let E_7 = event of getting a diamond. Then, $n(E_7) = 13$.

$$\therefore P(\text{getting a diamond}) = P(E_7) = \frac{n(E_7)}{n(S)} = \frac{13}{52} = \frac{1}{4}.$$

(viii) Let E_8 = event of getting a jack. Then, $n(E_8) = 4$.

$$\therefore P(\text{getting a jack}) = P(E_8) = \frac{n(E_8)}{n(S)} = \frac{4}{52} = \frac{1}{13}.$$

(ix) Let E_9 = event of getting a black card. Then, $n(E_9) = 26$.

$$\therefore P(\text{getting a black card}) = P(E_9) = \frac{n(E_9)}{n(S)} = \frac{26}{52} = \frac{1}{2}.$$

EXAMPLE 9 From a well-shuffled deck of 52 cards, a card is drawn at random. Find the probability that the card drawn is

(i) red and a king

(ii) either red or a king

SOLUTION Let S denote the sample space. Then, $n(S) = 52$.

(i) Let E_1 = event of drawing a red card which is a king.

We know that the number of red kings is 2. So, $n(E_1) = 2$.



$$\therefore P(\text{getting a red king}) = P(E_1) = \frac{n(E_1)}{n(S)} = \frac{2}{52} = \frac{1}{26}.$$

- (ii) Let E_2 = event of drawing a card which is either red or a king. There are 26 red cards (including 2 red kings) and there are 2 more kings.

$$\therefore n(E_2) = (26 + 2) = 28$$

$$\therefore P(\text{getting a red card or a king}) = P(E_2) = \frac{n(E_2)}{n(S)} = \frac{28}{52} = \frac{7}{13}.$$

EXAMPLE 10 A bag contains 9 red and 12 white balls. One ball is drawn at random. Find the probability that the ball drawn is red.

SOLUTION Total number of balls = $(9 + 12) = 21$.

Let S be the sample space. Then,

$$n(S) = \text{number of ways of selecting 1 ball out of 21} = 21.$$

Let E be the event of drawing a red ball. Then,

$$n(E) = \text{number of ways of selecting 1 red ball out of 9} = 9.$$

$$\therefore P(\text{getting a red ball}) = P(E) = \frac{n(E)}{n(S)} = \frac{9}{21} = \frac{3}{7}.$$

ODDS OF AN EVENT Let there be m outcomes favourable to an event E and n outcomes unfavourable to E . Then,

$$(i) \text{ odds in favour of } E = \frac{m}{n} \text{ or } (m : n)$$

$$(ii) \text{ odds against } E = \frac{n}{m} \text{ or } (n : m)$$

$$(iii) P(E) = \frac{m}{(m + n)}$$

EXAMPLE 11 The odds in favour of occurrence of an event are 5 : 12. Find the probability of the occurrence of this event.

SOLUTION Number of favourable outcomes = 5.

Number of unfavourable outcomes = 12.

Total number of outcomes = $(5 + 12) = 17$.

Let E be the event. Then,

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} = \frac{5}{17}.$$

EXAMPLE 12 If the probability of the occurrence of a certain event E is $\frac{3}{11}$, find (i) the odds in favour of its occurrence, and (ii) the odds against its occurrence.

SOLUTION Let there be m outcomes favourable to E and n outcomes unfavourable to E . Then,

$$P(E) = \frac{m}{m + n} \Rightarrow \frac{m}{m + n} = \frac{3}{11} \quad [\because P(E) = \frac{3}{11} \text{ (given)}]$$

$$\Rightarrow m = 3 \text{ and } n = 8.$$



(i) odds in favour of $E = \frac{m}{n} = \frac{3}{8}$.

(ii) odds against $E = \frac{n}{m} = \frac{8}{3}$.

PROBABILITY OF OCCURRENCE OF A COMPLEMENTARY EVENT

THEOREM 1 Let E be an event and $\bar{E}$ be the complement of E . Then, $P(\bar{E}) = 1 - P(E)$.

PROOF Let there be m outcomes favourable to E and n outcomes unfavourable to E . Then,

number of possible outcomes = $(m + n)$.

$$\therefore P(E) = \frac{m}{(m + n)} \text{ and } P(\bar{E}) = P(\text{not } E) = \frac{n}{(m + n)}$$

$$\Rightarrow P(E) + P(\bar{E}) = \left(\frac{m}{m + n} + \frac{n}{m + n} \right) = \frac{(m + n)}{(m + n)} = 1$$

$$\Rightarrow P(\bar{E}) = 1 - P(E).$$

Hence, $P(\bar{E}) = 1 - P(E)$.

NOTE There is an alternative proof of this theorem, which we will soon discuss.

EXAMPLE 13 If $\frac{3}{10}$ is the probability that an event will happen, what is the probability that it will not happen?

SOLUTION Let E be the event. Then,

$$P(E) = \frac{3}{10} \Rightarrow P(\bar{E}) = \{1 - P(E)\} = \left(1 - \frac{3}{10}\right) = \frac{7}{10}.$$

$$\text{Hence, } P(\text{not } E) = \frac{7}{10}.$$

EXAMPLE 14 Three dice are thrown together. Find the probability of getting a total of at least 6.

SOLUTION In throwing 3 dice together, the number of all possible outcomes is $(6 \times 6 \times 6) = 216$.

Let E = event of getting a total of at least 6.

Then, $\bar{E}$ = event of getting a total of less than 6

= event of getting a total of 3 or 4 or 5.

$$= \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1), (1, 1, 3), (1, 3, 1), \\ (3, 1, 1), (1, 2, 2), (2, 1, 2), (2, 2, 1)\}$$

$$\text{Now, } n(\bar{E}) = 10 \Rightarrow P(\text{not } E) = P(\bar{E}) = \frac{n(\bar{E})}{n(S)} = \frac{10}{216} = \frac{5}{108}.$$

$$\Rightarrow P(E) = 1 - P(\text{not } E) = \left(1 - \frac{5}{108}\right) = \frac{103}{108}.$$

Hence, the required probability is $\frac{103}{108}$.



THEOREM 2 For an event E ,

$$(i) \text{ odds in favour of } E = \frac{P(E)}{1 - P(E)}$$

$$(ii) \text{ odds against } E = \frac{1 - P(E)}{P(E)}$$

PROOF Let there be m outcomes favourable to E and n outcomes unfavourable to E .

Then, there are m outcomes favourable to E and n outcomes favourable to $\bar{E}$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{m}{(m+n)} \text{ and } P(\bar{E}) = \frac{n}{(m+n)}.$$

$$(i) \text{ Odds in favour of } E = \frac{m}{n} = \frac{\frac{m}{(m+n)}}{\frac{n}{(m+n)}} = \frac{P(E)}{P(\bar{E})} = \frac{P(E)}{1 - P(E)}.$$

$$(ii) \text{ Odds against } E = \frac{n}{m} = \frac{1 - P(E)}{P(E)}.$$

EXAMPLE 15 If the odds in favour of an event be $\frac{3}{5}$, find the probability of the occurrence of the event.

SOLUTION Let the given event be E and let $P(E) = x$. Then,

$$\text{odds in favour of } E = \frac{P(E)}{1 - P(E)}$$

$$\Leftrightarrow \frac{P(E)}{1 - P(E)} = \frac{3}{5} \Leftrightarrow \frac{x}{(1 - x)} = \frac{3}{5}$$

$$\Leftrightarrow 5x = 3 - 3x \Leftrightarrow 8x = 3 \Leftrightarrow x = \frac{3}{8}.$$

$$\therefore \text{ required probability} = \frac{3}{8}.$$

EXAMPLE 16 Two dice are thrown. Find (i) the odds in favour of getting the sum 5, and (ii) the odds against getting the sum 6.

SOLUTION Let S be the sample space. Then, $n(S) = 36$.

(i) Let E_1 be the event of getting the sum 5. Then,

$$E_1 = \{(1, 4), (2, 3), (3, 2), (4, 1)\} \Rightarrow n(E_1) = 4.$$

$$\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

$$\Rightarrow \text{odds in favour of } E_1 = \frac{P(E_1)}{1 - P(E_1)} = \frac{(\frac{1}{9})}{(1 - \frac{1}{9})} = \frac{1}{8}.$$



(ii) Let E_2 be the event of getting the sum 6. Then,

$$E_2 = \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\} \Rightarrow n(E_2) = 5.$$

$$\therefore P(E_2) = \frac{n(E_2)}{n(S)} = \frac{5}{36}$$

$$\Rightarrow \text{odds against } E_2 = \frac{1 - P(E_2)}{P(E_2)} = \frac{\left(1 - \frac{5}{36}\right)}{(5/36)} = \frac{31}{5}.$$

EXAMPLE 17 A card is drawn from a well-shuffled deck of 52 cards. Find (i) the odds in favour of getting a face card, and (ii) the odds against getting a spade.

SOLUTION When a card is drawn from a well-shuffled deck of 52 cards, the number of possible outcomes is 52.

Now, let S be the sample space. Then, $n(S) = 52$.

(i) Let E_1 be the event of getting a face card.

Since there are 16 face cards, we have $n(E_1) = 16$.

$$\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{16}{52} = \frac{4}{13}$$

$\Rightarrow$ odds in favour of getting a face card

$$= \frac{P(E_1)}{\{1 - P(E_1)\}} = \frac{(4/13)}{\left(1 - \frac{4}{13}\right)} = \frac{4}{9}.$$

(ii) Let E_2 be the event of getting a spade. Then, $n(E_2) = 13$.

$$\therefore P(E_2) = \frac{n(E_2)}{n(S)} = \frac{13}{52} = \frac{1}{4}$$

$$\begin{aligned} \Rightarrow \text{odds against getting a spade} &= \frac{\{1 - P(E_2)\}}{P(E_2)} \\ &= \frac{\left(1 - \frac{1}{4}\right)}{(1/4)} = \frac{3}{1}. \end{aligned}$$

EXAMPLE 18 Two cards are drawn at random from a pack of 52 cards. What is the probability that both the drawn cards are aces?

SOLUTION Let S be the sample space. Then,

$$\begin{aligned} n(S) &= \text{number of ways of selecting 2 cards out of 52} \\ &= {}^{52}C_2 = \frac{(52 \times 51)}{(2 \times 1)} = 1326. \end{aligned}$$

Let E = event that both the cards drawn are aces. Then,

$$\begin{aligned} n(E) &= \text{number of ways of drawing 2 aces out of 4} \\ &= {}^4C_2 = \frac{(4 \times 3)}{(2 \times 1)} = 6. \end{aligned}$$



$$P(\text{getting 2 aces}) = P(E) = \frac{n(E)}{n(S)} = \frac{6}{1326} = \frac{1}{221} .$$

EXERCISE 31A

1. A coin is tossed once. Find the probability of getting a tail.
2. A die is thrown. Find the probability of
 - (i) getting a 5
 - (ii) getting a 2 or a 3
 - (iii) getting an odd number
 - (iv) getting a prime number
 - (v) getting a multiple of 3
 - (vi) getting a number between 3 and 6
3. In a single throw of two dice, find the probability of
 - (i) getting a sum less than 6
 - (ii) getting a doublet of odd numbers
 - (iii) getting the sum as a prime number
4. In a single throw of two dice, find
 - (i) P (an odd number on the first die and a 6 on the second)
 - (ii) P (a number greater than 3 on each die)
 - (iii) P (a total of 10)
 - (iv) P (a total greater than 8)
 - (v) P (a total of 9 or 11)
5. A bag contains 4 white and 5 black balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is white.
6. An urn contains 9 red, 7 white and 4 black balls. A ball is drawn at random. Find the probability that the ball drawn is
 - (i) red
 - (ii) white
 - (iii) red or white
 - (iv) white or black
 - (v) not white
7. In a lottery, there are 10 prizes and 25 blanks. Find the probability of getting a prize.
8. If there are two children in a family, find the probability that there is at least one boy in the family.
9. Three unbiased coins are tossed once. Find the probability of getting
 - (i) exactly 2 tails
 - (ii) exactly one tail
 - (iii) at most 2 tails
 - (iv) at least 2 tails
 - (v) at most 2 tails or at least 2 heads[CBSE 2005]
10. In a single throw of two dice, determine the probability of not getting the same number on the two dice.
11. If a letter is chosen at random from the English alphabet, find the probability that the letter chosen is (i) a vowel, and (ii) a consonant.



12. A card is drawn at random from a well-shuffled pack of 52 cards. What is the probability that the card bears a number greater than 3 and less than 10?
13. Tickets numbered from 1 to 12 are mixed up together and then a ticket is withdrawn at random. Find the probability that the ticket has a number which is a multiple of 2 or 3.
14. What is the probability that an ordinary year has 53 Tuesdays?
15. What is the probability that a leap year has 53 Sundays?
16. What is the probability that in a group of two people, both will have the same birthday, assuming that there are 365 days in a year and no one has his/her birthday on 29th February?
17. Which of the following cannot be the probability of occurrence of an event?
 (i) 0 (ii) $\frac{-3}{4}$ (iii) $\frac{3}{4}$ (iv) $\frac{4}{3}$
18. If $\frac{7}{10}$ is the probability of occurrence of an event, what is the probability that it does not occur?
19. The odds in favour of the occurrence of an event are 8 : 13. Find the probability that the event will occur.
20. If the odds against the occurrence of an event be 4 : 7, find the probability of the occurrence of the event.
21. If $\frac{5}{14}$ is the probability of occurrence of an event, find
 (i) the odds in favour of its occurrence
 (ii) the odds against its occurrence
22. Two dice are thrown. Find
 (i) the odds in favour of getting the sum 6
 (ii) the odds against getting the sum 7
23. A combination lock on a suitcase has 3 wheels, each labelled with nine digits from 1 to 9. If an opening combination is a particular sequence of three digits with no repeats, what is the probability of a person guessing the right combination?
24. In a lottery, a person chooses six different numbers at random from 1 to 20. If these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game?
25. In a single throw of three dice, find the probability of getting (i) a total of 5
 (ii) a total of at most 5. [CBSE 2005]

ANSWERS (EXERCISE 31A)

1. $\frac{1}{2}$
2. (i) $\frac{1}{6}$ (ii) $\frac{1}{3}$ (iii) $\frac{1}{2}$ (iv) $\frac{1}{2}$ (v) $\frac{1}{3}$ (vi) $\frac{1}{3}$



3. (i) $\frac{5}{18}$ (ii) $\frac{1}{12}$ (iii) $\frac{5}{12}$ 4. (i) $\frac{1}{12}$ (ii) $\frac{1}{4}$ (iii) $\frac{1}{12}$ (iv) $\frac{5}{18}$ (v) $\frac{1}{6}$
5. $\frac{4}{9}$ 6. (i) $\frac{9}{20}$ (ii) $\frac{7}{20}$ (iii) $\frac{4}{5}$ (iv) $\frac{11}{20}$ (v) $\frac{13}{20}$ 7. $\frac{2}{7}$ 8. $\frac{3}{4}$
9. (i) $\frac{3}{8}$ (ii) $\frac{3}{8}$ (iii) $\frac{7}{8}$ (iv) $\frac{1}{2}$ (v) $\frac{7}{8}$ 10. $\frac{5}{6}$ 11. (i) $\frac{5}{26}$ (ii) $\frac{21}{26}$ 12. $\frac{6}{13}$
13. $\frac{2}{3}$ 14. $\frac{1}{7}$ 15. $\frac{2}{7}$ 16. $\frac{1}{365}$ 17. (ii) and (iv) 18. $\frac{3}{10}$ 19. $\frac{8}{21}$
20. $\frac{7}{11}$ 21. (i) 5 : 9 (ii) 9 : 5 22. (i) 5 : 31 (ii) 5 : 1 23. $\frac{1}{504}$
24. $\frac{1}{38760}$ 25. (i) $\frac{1}{36}$ (ii) $\frac{5}{108}$

HINTS TO SOME SELECTED QUESTIONS

7. There are 10 prizes and the total number of outcomes is 35.
8. $S = \{B_1 B_2, B_1 G_2, G_1 B_2, G_1 G_2\}$ and $E = \{B_1 G_2, G_1 B_2, B_1 B_2\}$.
 $\therefore$ required probability = $\frac{n(E)}{n(S)}$
10. $n(S) = (6 \times 6) = 36$.
 Let E = event of getting the same number on two dice
 or $E = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$.
 $\therefore P(E) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$
 $\Rightarrow P(\text{not getting the same number on the two dice}) = \left(1 - \frac{1}{6}\right) = \frac{5}{6}$.
11. There are 26 letters in the English alphabet, out of which there are 5 vowels and 21 consonants.
12. There are four suits in all, and the number of cards bearing a number greater than 3 and less than 10 in each suit is 6.
 $\therefore$ total number of favourable cases = $(6 \times 4) = 24$.
 Total number of cards = 52.
13. Clearly, $n(S) = 12$ and $E = \{2, 4, 6, 8, 10, 12, 3, 9\} \Rightarrow n(E) = 8$.
14. An ordinary year has 365 days, i.e., 52 weeks and 1 day.
 Now, 52 weeks have 52 Tuesdays and the remaining one day can be any of the 7 days.
 $\therefore$ required probability = probability of this day being a Tuesday = $\frac{1}{7}$.
15. A leap year has 366 days, and, therefore, 52 weeks and 2 days. These 2 days may be
 (i) Sunday and Monday or (ii) Monday and Tuesday
 or (iii) Tuesday and Wednesday or (iv) Wednesday and Thursday
 or (v) Thursday and Friday or (vi) Friday and Saturday
 or (vii) Saturday and Sunday.
 Thus, out of 7 possibilities, 2 favour the event that one of the two days is a Sunday.
 $\therefore$ required probability = $\frac{2}{7}$.



22. $n(S) = (6 \times 6) = 36$.

(i) Let E_1 = event of getting the sum 6
 $= \{(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)\}$

$\Rightarrow n(E_1) = 5$.

Thus, E_1 occurs in 5 ways and does not occur in $(36 - 5) = 31$ ways.

$\therefore$ odds in favour of getting the sum 6 = 5 : 31.

(ii) Let E_2 = event of getting the sum 7
 $= \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$

$\Rightarrow n(E_2) = 6$.

Thus, E_2 occurs in 6 ways and does not occur in $(36 - 6) = 30$ ways.

$\therefore$ odds against getting the sum 7 = 30 : 6 = 5 : 1.

23. Let us fill 3 places with different digits from 1 to 9.

Total number of ways = $(9 \times 8 \times 7) = 504$.

Thus, we have in all 504 combinations.

$P(\text{choosing one opening combination}) = \frac{1}{504}$.

24. $n(S)$ = number of ways of choosing 6 numbers out of 20
 $= {}^{20}C_6$.

There is a fixed set of 6 numbers, which can be chosen in 1 way only. Let E be the event of getting the fixed set of 6 numbers.

Then, $n(E) = 1$.

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{{}^{20}C_6}$.

25. $n(S) = (6 \times 6 \times 6) = 216$

(i) $E_1 = \{(1, 1, 3), (1, 3, 1), (3, 1, 1), (1, 2, 2), (2, 1, 2), (2, 2, 1)\}$

$\therefore P(E_1) = \frac{6}{216} = \frac{1}{36}$.

(ii) $E_2 = \{(1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1), (1, 1, 3), (1, 3, 1), (3, 1, 1), (1, 2, 2), (2, 1, 2), (2, 2, 1)\}$

$\therefore P(E_2) = \frac{10}{216} = \frac{5}{108}$.

RESULTS ON PROBABILITY

THEOREM 1 Let S be the sample space and let E be an event. Then,

(i) $P(E) \geq 0$ (ii) $P(\phi) = 0$ (iii) $P(S) = 1$

PROOF (i) Since E is an event, we have $E \subseteq S$.

$\therefore P(E) = \frac{n(E)}{n(S)} \geq 0$ [$\because n(E) \geq 0$].

(ii) $P(\phi) = \frac{n(\phi)}{n(S)} = \frac{0}{n(S)} = 0$.



$$(iii) P(S) = \frac{n(S)}{n(S)} = 1.$$

THEOREM 2 If E_1 and E_2 are mutually exclusive events,

$$(i) P(E_1 \text{ and } E_2) = 0, \text{ i.e., } P(E_1 \cap E_2) = 0$$

$$(ii) P(E_1 \text{ or } E_2) = P(E_1) + P(E_2), \text{ i.e., } P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

PROOF Let S be the sample space, and let E_1 and E_2 be two mutually exclusive events. Then, $E_1 \cap E_2 = \phi$.

$$(i) P(E_1 \text{ and } E_2) = P(E_1 \cap E_2) = P(\phi) = 0.$$

$$\begin{aligned} (ii) P(E_1 \text{ or } E_2) &= P(E_1 \cup E_2) \\ &= \frac{n(E_1 \cup E_2)}{n(S)} \\ &= \frac{n(E_1) + n(E_2)}{n(S)} \quad [\because E_1 \cap E_2 = \phi \Rightarrow n(E_1 \cup E_2) = n(E_1) + n(E_2)] \\ &= \frac{n(E_1)}{n(S)} + \frac{n(E_2)}{n(S)} = P(E_1) + P(E_2). \end{aligned}$$

$\therefore P(E_1 \text{ or } E_2) = P(E_1) + P(E_2)$, when E_1 and E_2 are mutually exclusive.

REMARK If $E_1, E_2, \dots, E_n$ are pairwise disjoint then

$$P(E_1 \cup E_2 \cup \dots \cup E_n) = P(E_1) + P(E_2) + \dots + P(E_n).$$

ADDITION THEOREM FOR TWO EVENTS

THEOREM 3 For any two events E_1 and E_2 ,

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2),$$

$$\text{i.e., } P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \text{ and } E_2).$$

PROOF Let S be the sample space. Then, $E_1 \subseteq S$ and $E_2 \subseteq S$.

From set theory, we know that

$$\begin{aligned} n(E_1 \cup E_2) &= n(E_1) + n(E_2) - n(E_1 \cap E_2) \\ \Rightarrow \frac{n(E_1 \cup E_2)}{n(S)} &= \frac{n(E_1) + n(E_2) - n(E_1 \cap E_2)}{n(S)} \\ &= \frac{n(E_1)}{n(S)} + \frac{n(E_2)}{n(S)} - \frac{n(E_1 \cap E_2)}{n(S)} \\ \Rightarrow P(E_1 \cup E_2) &= P(E_1) + P(E_2) - P(E_1 \cap E_2) \\ \Rightarrow P(E_1 \text{ or } E_2) &= P(E_1) + P(E_2) - P(E_1 \text{ and } E_2). \end{aligned}$$

REMARK If E_1 and E_2 are mutually exclusive events then $E_1 \cap E_2 = \phi$, and so $P(E_1 \cap E_2) = P(\phi) = 0$.

So, in this case, $P(E_1 \cup E_2) = P(E_1) + P(E_2)$.



ADDITION THEOREM FOR THREE EVENTS

THEOREM 4 For any three events E_1, E_2, E_3 ,

$$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_2 \cap E_3) - P(E_3 \cap E_1) + P(E_1 \cap E_2 \cap E_3).$$

PROOF We have

$$\begin{aligned} & P(E_1 \cup E_2 \cup E_3) \\ &= P[(E_1 \cup E_2) \cup E_3] \\ &= P(E_1 \cup E_2) + P(E_3) - P[(E_1 \cup E_2) \cap E_3] \\ & \quad \text{[by the addition theorem for two events]} \\ &= P(E_1) + P(E_2) - P(E_1 \cap E_2) + P(E_3) - P[(E_1 \cap E_3) \cup (E_2 \cap E_3)] \\ & \quad \text{[by the addition theorem on } P(E_1 \cup E_2)] \\ &= P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - [P(E_1 \cap E_3) + P(E_2 \cap E_3) \\ & \quad - P(E_1 \cap E_3 \cap E_2 \cap E_3)] \\ &= P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_1 \cap E_3) - P(E_2 \cap E_3) \\ & \quad + P(E_1 \cap E_2 \cap E_3) \end{aligned}$$

$$\text{Hence, } P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_2 \cap E_3) - P(E_3 \cap E_1) + P(E_1 \cap E_2 \cap E_3).$$

THEOREM 5 If E_1 and E_2 are two mutually exclusive exhaustive events then $P(E_1) + P(E_2) = 1$.

PROOF Let S be the sample space. Then, E_1 and E_2 being mutually exclusive exhaustive events, we have

$$\begin{aligned} & E_1 \cap E_2 = \phi \text{ and } E_1 \cup E_2 = S. \\ \therefore P(E_1) + P(E_2) &= P(E_1 \cup E_2) & [\because E_1 \cap E_2 = \phi] \\ &= P(S) = 1 & [\because E_1 \cup E_2 = S]. \\ \therefore (E_1 \cap E_2) = \phi \text{ and } (E_1 \cup E_2) = S &\Rightarrow P(E_1) + P(E_2) = 1. \end{aligned}$$

THEOREM 6 For any event E , $P(\bar{E}) = 1 - P(E)$.

PROOF Let S be the sample space.

$$\begin{aligned} & \text{Now, } (E \cap \bar{E}) = \phi \text{ and } (E \cup \bar{E}) = S \\ \Rightarrow P(E) + P(\bar{E}) &= P(E \cup \bar{E}) & [\because E \cap \bar{E} = \phi] \\ &= P(S) = 1 & [\because E \cup \bar{E} = S]. \end{aligned}$$

$$\text{Hence, } P(\bar{E}) = 1 - P(E).$$

NOTE For an alternative proof of this theorem, see P-17.

THEOREM 7 For any two compatible events E_1 and E_2 ,

$$P(E_1 - E_2) = P(E_1) - P(E_1 \cap E_2).$$

PROOF Let E_1 and E_2 be two compatible events.

$$\text{Then, } E_1 \cap E_2 \neq \phi.$$



From the adjoining Venn diagram, we have

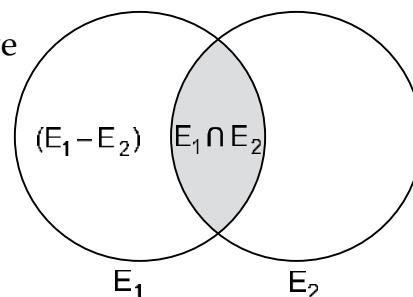
$$(E_1 - E_2) \cap (E_1 \cap E_2) = \phi, \quad \dots (i)$$

$$\text{and } (E_1 - E_2) \cup (E_1 \cap E_2) = E_1. \quad \dots (ii)$$

$$\therefore P(E_1) = P[(E_1 - E_2) \cup (E_1 \cap E_2)]$$

$$= P(E_1 - E_2) + P(E_1 \cap E_2)$$

$$\Rightarrow P(E_1 - E_2) = P(E_1) - P(E_1 \cap E_2).$$



THEOREM 8 If E_1 and E_2 be two events such that $E_1 \subseteq E_2$ then $P(E_1) \leq P(E_2)$.

PROOF Let $E_1 \subseteq E_2$. Then,

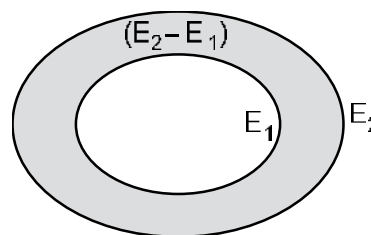
$$E_1 \cap (E_2 - E_1) = \phi \text{ and } E_1 \cup (E_2 - E_1) = E_2.$$

$$\therefore P(E_2) = P[E_1 \cup (E_2 - E_1)]$$

$$= P(E_1) + P(E_2 - E_1)$$

$$[\because E_1 \cap (E_2 - E_1) = \phi].$$

$$\therefore P(E_1) \leq P(E_2) \quad [\because P(E_2 - E_1) \geq 0].$$



THEOREM 9 For any event E , $0 \leq P(E) \leq 1$.

PROOF Let S be the sample space and let E be an event.

Then, $\phi \subseteq E$ and $E \subseteq S$

$$\Rightarrow P(\phi) \leq P(E) \text{ and } P(E) \leq P(S)$$

$$\Rightarrow P(\phi) \leq P(E) \leq P(S)$$

$$\Rightarrow 0 \leq P(E) \leq 1$$

$$[\because P(\phi) = 0 \text{ and } P(S) = 1].$$

Hence, $0 \leq P(E) \leq 1$, for every event E .

SOLVED EXAMPLES

EXAMPLE 1 If E_1 and E_2 are two events associated with a random experiment such that $P(E_2) = 0.35$, $P(E_1 \text{ or } E_2) = 0.85$ and $P(E_1 \text{ and } E_2) = 0.15$, find $P(E_1)$.

SOLUTION Let $P(E_1) = x$. Then,

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \text{ and } E_2)$$

$$\Rightarrow 0.85 = x + 0.35 - 0.15$$

$$\Rightarrow x = (0.85 - 0.35 + 0.15) = 0.65.$$

Hence, $P(E_1) = 0.65$.

EXAMPLE 2 Two dice are tossed together. Find the probability of getting a doublet or a total of 6. **[CBSE 2004C]**

SOLUTION When two dice are thrown together, there are (6×6) outcomes. Let S be the sample space. Then, $n(S) = 36$.

Let E_1 = event of getting a doublet,

and E_2 = event of getting a total of 6.



Then, $E_1 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$

and $E_2 = \{(2, 4), (3, 3), (4, 2)\}$.

$$\therefore (E_1 \cap E_2) = \{(3, 3)\}.$$

Thus, $n(E_1) = 6$, $n(E_2) = 3$ and $n(E_1 \cap E_2) = 1$.

$$\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}, P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

$$\text{and } P(E_1 \cap E_2) = \frac{n(E_1 \cap E_2)}{n(S)} = \frac{1}{36}.$$

$$\begin{aligned} \therefore P(\text{getting a doublet or a total of 6}) &= P(E_1 \text{ or } E_2) = P(E_1 \cup E_2) \\ &= P(E_1) + P(E_2) - P(E_1 \cap E_2) \quad [\text{by the addition theorem}] \\ &= \left(\frac{1}{6} + \frac{1}{12} - \frac{1}{36} \right) = \frac{8}{36} = \frac{2}{9}. \end{aligned}$$

Hence, the required probability is $\frac{2}{9}$.

EXAMPLE 3 In a single throw of two dice, find the probability that neither a doublet nor a total of 10 will appear. [CBSE 1998, 2003]

SOLUTION Let S be the sample space. Then, $n(S) = 36$.

Let E_1 = event that a doublet appears,

and E_2 = event of getting a total of 10.

Then, $E_1 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$,

and $E_2 = \{(4, 6), (5, 5), (6, 4)\}$.

$$\therefore (E_1 \cap E_2) = \{(5, 5)\}.$$

Thus, $n(E_1) = 6$, $n(E_2) = 3$ and $n(E_1 \cap E_2) = 1$.

$$\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}, P(E_2) = \frac{n(E_2)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

$$\text{and } P(E_1 \cap E_2) = \frac{n(E_1 \cap E_2)}{n(S)} = \frac{1}{36}.$$

$$\begin{aligned} \therefore P(\text{getting a doublet or a total of 10}) &= P(E_1 \text{ or } E_2) = P(E_1 \cup E_2) \\ &= P(E_1) + P(E_2) - P(E_1 \cap E_2) \\ &= \left(\frac{1}{6} + \frac{1}{12} - \frac{1}{36} \right) = \frac{8}{36} = \frac{2}{9}. \end{aligned}$$



$$\begin{aligned}
\therefore P(\text{getting neither a doublet nor a total of 10}) \\
&= P(\bar{E}_1 \text{ and } \bar{E}_2) = P(\bar{E}_1 \cap \bar{E}_2) \\
&= P(\overline{E_1 \cup E_2}) = 1 - P(E_1 \cup E_2) = \left(1 - \frac{2}{9}\right) = \frac{7}{9}.
\end{aligned}$$

Hence, the required probability is $\frac{7}{9}$.

EXAMPLE 4 A natural number is chosen at random from among the first 500. What is the probability that the number so chosen is divisible by 3 or 5?

SOLUTION Let S be the sample space. Then, clearly $n(S) = 500$.

Let E_1 = event of getting a number divisible by 3,

and E_2 = event of getting a number divisible by 5. Then,

$$\begin{aligned}
(E_1 \cap E_2) &= \text{event of getting a number divisible by both 3 and 5} \\
&= \text{event of getting a number divisible by 15.}
\end{aligned}$$

$$\therefore E_1 = \{3, 6, 9, \dots, 495, 498\}, E_2 = \{5, 10, 15, \dots, 495, 500\}$$

$$\text{and } (E_1 \cap E_2) = \{15, 30, 45, \dots, 495\}.$$

$$\therefore n(E_1) = \left(\frac{498}{3}\right) = 166, n(E_2) = \left(\frac{500}{5}\right) = 100$$

$$\text{and } n(E_1 \cap E_2) = \left(\frac{495}{15}\right) = 33.$$

$$\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{166}{500} = \frac{83}{250}, P(E_2) = \frac{n(E_2)}{n(S)} = \frac{100}{500} = \frac{1}{5}$$

$$\text{and } P(E_1 \cap E_2) = \frac{n(E_1 \cap E_2)}{n(S)} = \frac{33}{500}.$$

$$\therefore P(\text{the chosen number is divisible by 3 or 5})$$

$$= P(E_1 \text{ or } E_2) = P(E_1 \cup E_2)$$

$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \left(\frac{83}{250} + \frac{1}{5} - \frac{33}{500}\right) = \frac{233}{500}.$$

Hence, the required probability is $\frac{233}{500}$.

EXAMPLE 5 A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability of its being a spade or a king.

SOLUTION Let S be the sample space. Then, $n(S) = 52$.

Let E_1 = event of getting a spade,

and E_2 = event of getting a king.

Then, $(E_1 \cap E_2)$ = event of getting a king of spades.

Clearly, $n(E_1) = 13$, $n(E_2) = 4$ and $n(E_1 \cap E_2) = 1$.



$$\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{13}{52} = \frac{1}{4}, P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{52} = \frac{1}{13}$$

$$\text{and } P(E_1 \cap E_2) = \frac{n(E_1 \cap E_2)}{n(S)} = \frac{1}{52}.$$

$$\begin{aligned} \therefore P(\text{getting a spade or a king}) &= P(E_1 \text{ or } E_2) = P(E_1 \cup E_2) \\ &= P(E_1) + P(E_2) - P(E_1 \cap E_2) \text{ [by the addition theorem for two events]} \end{aligned}$$

$$= \left(\frac{1}{4} + \frac{1}{13} - \frac{1}{52} \right) = \frac{16}{52} = \frac{4}{13}.$$

Hence, the required probability is $\frac{4}{13}$.

EXAMPLE 6 Two cards are drawn at random from a well-shuffled pack of 52 cards. What is the probability that either both are red or both are kings?

SOLUTION Let S be the sample space. Then,

$$\begin{aligned} n(S) &= \text{number of ways of drawing 2 cards out of 52} \\ &= {}^{52}C_2. \end{aligned}$$

Let E_1 = event that both are red cards,

and E_2 = event that both are kings.

Then, $(E_1 \cap E_2)$ = event of getting 2 red kings.

$$\therefore n(E_1) = \text{number of ways of drawing 2 red cards out of 26 red cards} = {}^{26}C_2.$$

$$\begin{aligned} n(E_2) &= \text{number of ways of drawing 2 kings out of 4 kings} \\ &= {}^4C_2. \end{aligned}$$

$$\therefore n(E_1 \cap E_2) = \text{number of ways of drawing 2 red kings out of 2 red kings} = {}^2C_2 = 1.$$

$$\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{{}^{26}C_2}{{}^{52}C_2}; P(E_2) = \frac{n(E_2)}{n(S)} = \frac{{}^4C_2}{{}^{52}C_2}$$

$$\text{and } P(E_1 \cap E_2) = \frac{n(E_1 \cap E_2)}{n(S)} = \frac{1}{{}^{52}C_2}.$$

$$\therefore P(\text{drawing both red cards or both kings})$$

$$= P(E_1 \text{ or } E_2) = P(E_1 \cup E_2)$$

$$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \left(\frac{{}^{26}C_2}{{}^{52}C_2} + \frac{{}^4C_2}{{}^{52}C_2} - \frac{1}{{}^{52}C_2} \right) = \frac{({}^{26}C_2 + {}^4C_2 - 1)}{{}^{52}C_2}$$



$$= \frac{(325 + 6 - 1)}{1326} = \frac{330}{1326} = \frac{55}{221}.$$

Hence, the required probability is $\frac{55}{221}$.

EXAMPLE 7 A box contains 100 bolts and 50 nuts. It is given that 50% bolts and 50% nuts are rusted. Two objects are selected from the box at random. Find the probability that either both are bolts or both are rusted. [CBSE 2003C]

SOLUTION Total number of objects = $(100 + 50) = 150$.

Let S be the sample space. Then,

$$\begin{aligned} n(S) &= \text{number of ways of selecting 2 objects out of 150} \\ &= {}^{150}C_2. \end{aligned}$$

Number of rusted objects

$$= (50\% \text{ of } 100) + (50\% \text{ of } 50) = (50 + 25) = 75.$$

Let E_1 = event of selecting 2 bolts out of 100 bolts,

and E_2 = event of selecting 2 rusted objects out of 75 rusted objects.

$\therefore (E_1 \cap E_2)$ = event of selecting 2 rusted bolts out of 50 rusted bolts

$$\begin{aligned} \therefore n(E_1) &= \text{number of ways of selecting 2 bolts out of 100} \\ &= {}^{100}C_2. \end{aligned}$$

$$\begin{aligned} \therefore n(E_2) &= \text{number of ways of selecting 2 rusted objects out of 75} \\ &= {}^{75}C_2. \end{aligned}$$

$$\begin{aligned} \therefore n(E_1 \cap E_2) &= \text{number of ways of selecting 2 rusted bolts out of 50} \\ &= {}^{50}C_2. \end{aligned}$$

$$\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{{}^{100}C_2}{{}^{150}C_2}, P(E_2) = \frac{n(E_2)}{n(S)} = \frac{{}^{75}C_2}{{}^{150}C_2}$$

$$\text{and } P(E_1 \cap E_2) = \frac{n(E_1 \cap E_2)}{n(S)} = \frac{{}^{50}C_2}{{}^{150}C_2}.$$

$P(\text{selecting both bolts or both rusted objects})$

$$\begin{aligned} &= P(E_1 \text{ or } E_2) = P(E_1 \cup E_2) \\ &= P(E_1) + P(E_2) - P(E_1 \cap E_2) \\ &= \frac{{}^{100}C_2}{{}^{150}C_2} + \frac{{}^{75}C_2}{{}^{150}C_2} - \frac{{}^{50}C_2}{{}^{150}C_2} = \frac{({}^{100}C_2 + {}^{75}C_2 - {}^{50}C_2)}{{}^{150}C_2} \\ &= \frac{(4950 + 2775 - 1225)}{11175} = \frac{6500}{11175} = \frac{260}{447} = 0.58. \end{aligned}$$

Hence, the required probability is 0.58.



EXAMPLE 8 If E_1 and E_2 are two events such that $P(E_1) = 0.5$, $P(E_2) = 0.3$ and $P(E_1 \text{ and } E_2) = 0.1$, find

- | | |
|-------------------------------------|--|
| (i) $P(E_1 \text{ or } E_2)$ | (ii) $P(E_1 \text{ but not } E_2)$ |
| (iii) $P(E_2 \text{ but not } E_1)$ | (iv) $P(\text{neither } E_1 \text{ nor } E_2)$ |

SOLUTION We have

$$P(E_1) = 0.5, P(E_2) = 0.3 \text{ and } P(E_1 \cap E_2) = P(E_1 \text{ and } E_2) = 0.1.$$

$$\therefore P(\bar{E}_1) = \{1 - P(E_1)\} = (1 - 0.5) = 0.5,$$

$$\text{and } P(\bar{E}_2) = \{1 - P(E_2)\} = (1 - 0.3) = 0.7.$$

Thus, we have

- (i) $P(E_1 \text{ or } E_2) = P(E_1 \cup E_2)$
 $= P(E_1) + P(E_2) - P(E_1 \cap E_2)$
 $= (0.5 + 0.3 - 0.1) = 0.7.$
- (ii) $P(E_1 \text{ but not } E_2) = P(E_1 \cap \bar{E}_2)$
 $= P(E_1) - P(E_1 \cap E_2)$
 $= (0.5 - 0.1) = 0.4.$
- (iii) $P(E_2 \text{ but not } E_1) = P(E_2 \cap \bar{E}_1)$
 $= P(E_2) - P(E_2 \cap E_1)$
 $= P(E_2) - P(E_1 \cap E_2) = (0.3 - 0.1) = 0.2.$
- (iv) $P(\text{neither } E_1 \text{ nor } E_2) = P(\text{not } E_1 \text{ and not } E_2)$
 $= P(\bar{E}_1 \text{ and } \bar{E}_2) = P(\bar{E}_1 \cap \bar{E}_2)$
 $= P(\overline{E_1 \cup E_2}) = 1 - P(E_1 \cup E_2)$
 $= 1 - P(E_1 \text{ or } E_2)$
 $= (1 - 0.7) = 0.3 \quad [\text{using (i)}].$

EXAMPLE 9 The probability that at least one of the events E_1 and E_2 occurs is 0.6. If the probability of the simultaneous occurrence of E_1 and E_2 is 0.2, find $P(\bar{E}_1) + P(\bar{E}_2)$.

SOLUTION Given, $P(E_1 \cup E_2) = 0.6$ and $P(E_1 \cap E_2) = 0.2$.

$$\therefore P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$\Rightarrow P(E_1) + P(E_2) = P(E_1 \cup E_2) + P(E_1 \cap E_2) = (0.6 + 0.2) = 0.8$$

$$\Rightarrow P(E_1) + P(E_2) = 0.8$$

$$\Rightarrow \{1 - P(\bar{E}_1)\} + \{1 - P(\bar{E}_2)\} = 0.8$$

$$\Rightarrow P(\bar{E}_1) + P(\bar{E}_2) = (2 - 0.8) = 1.2.$$

Hence, $P(\bar{E}_1) + P(\bar{E}_2) = 1.2$.

EXAMPLE 10 The probabilities of the occurrences of two events E_1 and E_2 are 0.25 and 0.50 respectively. The probability of their simultaneous occurrence is 0.14. Find the probability that neither E_1 nor E_2 occurs.



SOLUTION Given, $P(E_1) = 0.25$, $P(E_2) = 0.50$

and $P(E_1 \text{ and } E_2) = P(E_1 \cap E_2) = 0.14$.

$\therefore P(\bar{E}_1) = 1 - P(E_1) = (1 - 0.25) = 0.75$

and $P(\bar{E}_2) = 1 - P(E_2) = (1 - 0.50) = 0.50$.

$\therefore P(\text{neither } E_1 \text{ nor } E_2)$

$$= P(\text{not } E_1 \text{ and not } E_2) = P(\bar{E}_1 \text{ and } \bar{E}_2) = P(\bar{E}_1 \cap \bar{E}_2)$$

$$= P(\overline{E_1 \cup E_2}) = \{1 - P(E_1 \cup E_2)\}$$

$$= 1 - \{P(E_1) + P(E_2) - P(E_1 \cap E_2)\}$$

$$= 1 - (0.25 + 0.50 - 0.14) = (1 - 0.61) = 0.39.$$

Hence, the required probability is 0.39.

EXAMPLE 11 A card is drawn from a deck of 52 cards. Find the probability of getting a king or a heart or a red card.

SOLUTION Let S be the sample space. Then, $n(S) = 52$.

Let E_1 , E_2 and E_3 be the events of getting a king, a heart and a red card respectively. Then,

$$n(E_1) = 4, n(E_2) = 13 \text{ and } n(E_3) = 26.$$

$(E_1 \cap E_2)$ = event of getting a king of hearts;

$(E_2 \cap E_3)$ = event of getting a heart [$\because$ a heart is a red card also];

$(E_3 \cap E_1)$ = event of getting a red king;

and $(E_1 \cap E_2 \cap E_3)$ = event of getting a king of hearts.

$$\therefore n(E_1 \cap E_2) = 1, n(E_2 \cap E_3) = 13, n(E_3 \cap E_1) = 2$$

$$\text{and } n(E_1 \cap E_2 \cap E_3) = 1.$$

$$\therefore P(E_1) = \frac{n(E_1)}{n(S)} = \frac{4}{52} = \frac{1}{13}; P(E_2) = \frac{n(E_2)}{n(S)} = \frac{13}{52} = \frac{1}{4};$$

$$P(E_3) = \frac{n(E_3)}{n(S)} = \frac{26}{52} = \frac{1}{2}; P(E_1 \cap E_2) = \frac{n(E_1 \cap E_2)}{n(S)} = \frac{1}{52};$$

$$P(E_2 \cap E_3) = \frac{n(E_2 \cap E_3)}{n(S)} = \frac{13}{52} = \frac{1}{4};$$

$$P(E_3 \cap E_1) = \frac{n(E_3 \cap E_1)}{n(S)} = \frac{2}{52} = \frac{1}{26}$$

$$\text{and } P(E_1 \cap E_2 \cap E_3) = \frac{n(E_1 \cap E_2 \cap E_3)}{n(S)} = \frac{1}{52}.$$

$\therefore P(\text{getting a king or a heart or a red card})$

$$= P(E_1 \text{ or } E_2 \text{ or } E_3) = P(E_1 \cup E_2 \cup E_3)$$

$$= P(E_1) + P(E_2) + P(E_3) - P(E_1 \cap E_2) - P(E_2 \cap E_3) - P(E_3 \cap E_1) + P(E_1 \cap E_2 \cap E_3)$$



$$= \left(\frac{1}{13} + \frac{1}{4} + \frac{1}{2} - \frac{1}{52} - \frac{1}{4} - \frac{1}{26} + \frac{1}{52} \right) = \frac{28}{52} = \frac{7}{13}.$$

Hence, the required probability is $\frac{7}{13}$.

EXERCISE 31B

1. If A and B are two events associated with a random experiment for which $P(A) = 0.60$, $P(A \text{ or } B) = 0.85$ and $P(A \text{ and } B) = 0.42$, find $P(B)$. [CBSE 1998]
2. Let A and B be two events associated with a random experiment for which $P(A) = 0.4$, $P(B) = 0.5$ and $P(A \text{ or } B) = 0.6$. Find $P(A \text{ and } B)$.
3. In a random experiment, let A and B be events such that $P(A \text{ or } B) = 0.7$, $P(A \text{ and } B) = 0.3$ and $P(\bar{A}) = 0.4$. Find $P(B)$.
4. If A and B are two events associated with a random experiment such that $P(A) = 0.25$, $P(B) = 0.4$ and $P(A \text{ or } B) = 0.5$, find the values of
(i) $P(A \text{ and } B)$ (ii) $P(A \text{ and } \bar{B})$
5. If A and B be two events associated with a random experiment such that $P(A) = 0.3$, $P(B) = 0.2$ and $P(A \cap B) = 0.1$, find
(i) $P(\bar{A} \cap B)$ (ii) $P(A \cap \bar{B})$
6. If A and B are two mutually exclusive events such that $P(A) = (1/2)$ and $P(B) = (1/3)$, find $P(A \text{ or } B)$.
7. Let A and B be two mutually exclusive events of a random experiment such that $P(\text{not } A) = 0.65$ and $P(A \text{ or } B) = 0.65$, find $P(B)$.
8. A , B , C are three mutually exclusive and exhaustive events associated with a random experiment.
If $P(B) = (3/2) P(A)$ and $P(C) = (1/2) P(B)$, find $P(A)$.
9. The probability that a company executive will travel by plane is $(2/5)$ and that he will travel by train is $(1/3)$. Find the probability of his travelling by plane or train.
10. From a well-shuffled pack of 52 cards, a card is drawn at random. Find the probability of its being a king or a queen.
11. From a well-shuffled pack of cards, a card is drawn at random. Find the probability of its being either a queen or a heart.
12. A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability of its being a spade or a king.
13. A number is chosen from the numbers 1 to 100. Find the probability of its being divisible by 4 or 6.
14. A die is thrown twice. What is the probability that at least one of the two throws comes up with the number 4?



15. Two dice are tossed once. Find the probability of getting an even number on the first die or a total of 8. [CBSE 2004]
16. Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is neither divisible by 3 nor by 4.
17. In a class, 30% of the students offered mathematics, 20% offered chemistry and 10% offered both. If a student is selected at random, find the probability that he has offered mathematics or chemistry.
18. The probability that Hemant passes in English is $(\frac{2}{3})$ and the probability that he passes in Hindi is $(\frac{5}{9})$. If the probability of his passing both the subjects is $(\frac{2}{5})$, find the probability that he will pass in at least one of these subjects.
19. The probability that a person will get an electrification contract is $(\frac{2}{5})$ and the probability that he will not get a plumbing contract is $(\frac{4}{7})$. If the probability of getting at least one contract is $(\frac{2}{3})$, what is the probability that he will get both? [CBSE 2005C]
20. The probability that a patient visiting a dentist will have a tooth extracted is 0.06, the probability that he will have a cavity filled is 0.2, and the probability that he will have a tooth extracted or a cavity filled is 0.23. What is the probability that he will have a tooth extracted as well as a cavity filled?
21. In a town of 6000 people, 1200 are over 50 years old and 2000 are females. It is known that 30% of the females are over 50 years. What is the probability that a randomly chosen individual from the town is either female or over 50 years?

ANSWERS (EXERCISE 31B)

1. 0.67 2. 0.3 3. 0.4 4. (i) 0.15 (ii) 0.1 5. (i) 0.1 (ii) 0.2 6. $\frac{5}{6}$
7. 0.3 8. $\frac{4}{13}$ 9. $\frac{11}{15}$ 10. $\frac{2}{13}$ 11. $\frac{4}{13}$ 12. $\frac{4}{13}$ 13. $\frac{33}{100}$ 14. $\frac{11}{36}$
15. $\frac{5}{9}$ 16. $\frac{4}{9}$ 17. $\frac{2}{5}$ 18. $\frac{37}{45}$ 19. $\frac{17}{105}$ 20. 0.03 21. $\frac{13}{30}$

HINTS TO SOME SELECTED QUESTIONS

4. (ii) $P(A \text{ and } \bar{B}) = P(A \cap \bar{B}) = P(A) - P(A \cap B)$.

5. (i) $P(\bar{A} \cap B) = P(B) - P(A \cap B)$.

6. $A \cap B = \phi \Rightarrow P(A \cup B) = P(A) + P(B)$.

8. A, B, C being mutually exclusive exhaustive events, we have

$$P(A) + P(B) + P(C) = 1 \Rightarrow P(A) + \frac{3}{2}P(A) + \frac{1}{2} \cdot \left\{ \frac{3}{2}P(A) \right\} = 1$$

$$\Rightarrow \frac{13}{4} \cdot P(A) = 1 \Rightarrow P(A) = \frac{4}{13}$$



9. The events of travelling by plane and that by train are mutually exclusive.
 $\therefore P(E_1 \cup E_2) = P(E_1) + P(E_2)$.
10. The probability of getting a king and that of getting a queen are mutually exclusive.
 $\therefore P(\text{a king or a queen}) = P(\text{a king}) + P(\text{a queen}) = \left(\frac{4}{52} + \frac{4}{52} \right)$.
11. Clearly, $n(S) = 52$.
 Let E_1 = event of getting a queen,
 and E_2 = event of getting a heart.
 Then, $E_1 \cap E_2$ = event of getting a queen of hearts.
 $\therefore n(E_1) = 4, n(E_2) = 13$ and $n(E_1 \cap E_2) = 1$.
 Now, use $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$.
13. Here, $S = \{1, 2, 3, 4, \dots, 99, 100\} \Rightarrow n(S) = 100$.
 Let E_1 = event of getting a number divisible by 4.
 And, E_2 = event of getting a number divisible by 6.
 Then, $E_1 \cap E_2$ = event of getting a number divisible by both 4 and 6, i.e., divisible by 12 (LCM of 4 and 6).
 $\therefore E_1 = \{4, 8, 12, \dots, 100\} \Rightarrow n(E_1) = 25$.
 $E_2 = \{6, 12, 18, \dots, 96\} \Rightarrow n(E_2) = 16$.
 $(E_1 \cap E_2) = \{12, 24, 36, \dots, 96\} \Rightarrow n(E_1 \cap E_2) = 8$.
 Now, use $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$.
14. Clearly, $n(S) = (6 \times 6) = 36$.
 Let E_1 = event that the first throw shows a 4,
 and E_2 = event that the second throw shows a 4.
 Then, $(E_1 \cap E_2)$ = event that each throw shows a 4.
 $\therefore E_1 = \{(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)\}$,
 and $E_2 = \{(1, 4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4)\}$.
 $\therefore (E_1 \cap E_2) = \{(4, 4)\}$.
 Now, use $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$.
15. Clearly, $n(S) = (6 \times 6) = 36$.
 Let E_1 = event of getting an even number on the first die,
 and E_2 = event of getting a total of 8 on both dice. Then,
 $E_1 = \{(2, 1), (2, 2), \dots, (2, 6), (4, 1), (4, 2), \dots, (4, 6), (6, 1), (6, 2), \dots, (6, 6)\}$,
 $E_2 = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$.
 $\therefore (E_1 \cap E_2) = \{(2, 6), (4, 4), (6, 2)\}$.
 $\therefore n(E_1) = 18, n(E_2) = 5$ and $n(E_1 \cap E_2) = 3$.
 Now, use $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$.
16. Clearly, $n(S) = (6 \times 6) = 36$.
 Let E_1 = event that the sum of numbers on two faces is divisible by 3,
 and E_2 = event that the sum of numbers on two faces is divisible by 4.
 Then, $E_1 \cap E_2$ = event that the sum of the numbers on two faces is divisible by both 3 and 4, i.e., by 12.



$$\therefore E_1 = \{(1, 2), (1, 5), (2, 1), (2, 4), (3, 3), (3, 6), (4, 2), (4, 5), (5, 1), (5, 4), (6, 3), (6, 6)\},$$

$$E_2 = \{(1, 3), (2, 2), (2, 6), (3, 1), (3, 5), (4, 4), (5, 3), (6, 2), (6, 6)\}.$$

$$\therefore E_1 \cap E_2 = \{(6, 6)\}.$$

$$\text{Thus, } n(E_1) = 12, n(E_2) = 9 \text{ and } n(E_1 \cap E_2) = 1.$$

$$\begin{aligned} \text{Required probability} &= P[(\text{not } E_1) \text{ and } (\text{not } E_2)] = P(\bar{E}_1 \cap \bar{E}_2) \\ &= P(\overline{E_1 \cup E_2}) = 1 - P(E_1 \cup E_2) \\ &= 1 - \{P(E_1) + P(E_2) - P(E_1 \cap E_2)\}. \end{aligned}$$

17. Here, $n(S) = 100, n(M) = 30, n(C) = 20$ and $n(M \cap C) = 10$.

$$\therefore P(M) = \frac{30}{100} = \frac{3}{10}; P(C) = \frac{20}{100} = \frac{1}{5} \text{ and } P(M \cap C) = \frac{10}{100} = \frac{1}{10}.$$

$$\text{Use } P(M \cup C) = P(M) + P(C) - P(M \cap C).$$

18. $P(E) = \frac{2}{3}, P(H) = \frac{5}{9}$ and $P(E \cap H) = \frac{2}{5}$. Find $P(E \cup H)$.

19. Let E_1 = event of getting an electrification contract,
and E_2 = event of getting a plumbing contract.

$$\text{Then, } P(E_1) = \frac{2}{5}, P(\text{not } E_2) = \frac{4}{7} \Rightarrow P(E_2) = \left(1 - \frac{4}{7}\right) = \frac{3}{7}.$$

$$\text{And, } P(E_1 \cup E_2) = P(E_1 \text{ or } E_2) = \frac{2}{3}.$$

$$\begin{aligned} \therefore \text{required probability} &= P(E_1 \cap E_2) \\ &= P(E_1) + P(E_2) - P(E_1 \cup E_2). \end{aligned}$$

20. $P(E_1) = 0.06, P(E_2) = 0.2$ and $P(E_1 \cup E_2) = 0.23$.

$$P(E_1 \cap E_2) = P(E_1) + P(E_2) - P(E_1 \cup E_2).$$

21. Let E_1 = event of person being a female,
and E_2 = event of person being 50 years old.

$$\text{Then, } n(E_1) = 2000, n(E_2) = 1200,$$

$$\text{and } n(E_1 \cap E_2) = (30\% \text{ of } 2000) = 600.$$

$$\text{Now, } n(E_1 \cup E_2) = n(E_1) + n(E_2) - n(E_1 \cap E_2) = 2600.$$

$$\therefore P(E_1 \cup E_2) = \frac{2600}{6000} = \frac{13}{30}.$$

